## MEMORANDUM

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To : COMPASS collaboration
Subject : Kaon multiplicity ratio at high $z$.

An evaluation of the $K^{+} / K^{-}$multiplicity ratio at high $z$ for the deuteron target in the DIS region is presented. The measured ratio cannot be accommodated within the formalism of LO pQCD . Arguments will be presented which support the same conclusion also in NLO pQCD.

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## 1 Introduction

COMPASS recently has published kaon multiplicities in [1]. The current analysis extends the kinematic region of the former analysis towards large $z$, where $z$ is the fraction of the virtual photon energy carried by a hadron. It should be stressed that a lot of work has been done for the analysis published in [1]. To fully capitalise on these work, whenever possible exactly the same cuts are used. Basically only a few differences are present: $i$ ) The $z$ range was extended towards high $z$. ii) The acceptance method used in [1] is not optimal when working at the edges of the phase space. While the core of the method is kept the same a special treatment of $z$ smearing is discussed in section 4.2. iii) As presented in October analysis meeting a problem with the RICH PID in the region of high $z$ and high momenta was discovered [3], [4]. The problem can be avoided using a stricter cut on the kaon likelihoods. While this is not an optimal solution from the point of view of statistical precision it provides the smallest change with respect to the published paper. iv) All 2006 data but W35 are used. v) All physics triggers are used and not only inclusive ones.

## 2 Motivation

So far the multiplicities in the high $z$ region were measured only in $e^{+} e^{-}$experiments. COMPASS is the 1st experiment which explores in detail this region in a SIDIS type of measurement.
The presented work concentrates on kaons. Indeed in case of $\pi$ the high $z$ region is dominated by the decay of diffractively produced $\rho^{0}$ meson to two pions, thus does not allow in an easy way to study pQCD in this region. In case of kaons the dominant vector meson is the $\phi$ meson. But due to its low mass w.r.t. $2 \cdot K$ mass threshold, kaons from its decay have very low transverse momenta in the $\phi$ centre of mass system. As a result in COMPASS we observe these kaons only in the region $0.3<z<0.7$. Heavier vector mesons have smaller cross-sections and their decays are dominated by multi-particle ones. Thus, the probability to obtain a kaon at high $z$ from diffractive processes is negligible. The high $z$ region is also less polluted by higher order processes like PGF and QCDC, e.g. the cross-section for $D^{*}$ production is concentrated around $z=0.5$ as presented in COMPASS paper [2].
In the present work we concentrate only on the ratio of the $K^{+} / K^{-}$, which is motivated by following considerations. First of all, we have a problem with semi-inclusive radiative corrections. These corrections should largely cancel in the ratio of multiplicities. Secondly, a lot of systematic uncertainties cancel in the ratio. Indeed in [1] we claim the correlated part of the systematic error is about $80 \%$ of the quoted uncertainty. Finally, analysis of the ratio is simplified as the DIS sample (and MC) are not needed. Yet, the strongest motivation for the study of kaon multiplicity ratio comes from pQCD.
While pQCD cannot predict values of fragmentation functions, i.e. multiplicities of $K^{+}$or $K^{-}$, it can make some predictions concerning the ratio of the two. Namely, in LO pQDC,assuming as usually that we deal with 3 fragmentation functions: $D_{\text {fav }}, D_{s t r} D_{u n f}$, the kaon multiplicity ratio, for the simplicity written for the proton target, is the following:

$$
\begin{equation*}
\frac{M^{K^{+}}}{M^{K^{-}}}=\frac{4 u D_{f a v}+(4 \bar{u}+d+\bar{d}+s) D_{u n f}+\bar{s} D_{s t r}}{4 \bar{u} D_{f a v}+(4 u+d+\bar{d}+\bar{s}) D_{u n f}+s D_{s t r}} \tag{1}
\end{equation*}
$$

In most of the QCD fits, for the kaon case $D_{\text {unf }}$ is already small for $z=0.4$, thus can be safely neglected in the high $z$ region. This simplifies the ratio to:

$$
\begin{equation*}
\frac{M^{K^{+}}}{M^{K^{-}}}=\frac{4 u D_{\text {fav }}+\bar{s} D_{s t r}}{4 \bar{u} D_{\text {fav }}+s D_{s t r}}=\frac{4 \frac{u}{\bar{u}}+R}{4+R} \tag{2}
\end{equation*}
$$

where $R=\frac{s}{\bar{u}} \frac{D_{s t r}}{D_{\text {fav }}}$ and $s=\bar{s}$ was assumed. Since R can be only positive one has a simple limit:

$$
\begin{equation*}
\frac{M^{K^{+}}}{M^{K^{-}}}<\frac{u}{\bar{u}} \text { or } \frac{u+d}{\bar{u}+\bar{d}} \text { for deuteron. } \tag{3}
\end{equation*}
$$

The comparison of the limit with the DSS fit is presented in Fig. 1. In the same figure results for the LUND string model are presented. In case of the Lund model there are some additional $K^{+}$produced at very high values of $z$, even above 1.0, as the LUND MC is not too strict concerning energy conservation. However, in both cases the resulting ratios are in most of the phase space well below the limit derived above.


Figure 1: Comparison of LO QCD prediction of the $K^{+} / K^{-}$limit, i.e. $(u+d) /(\bar{u}+\bar{d})$ for $x=0.03$ and $Q^{2}=1.6(\mathrm{GeV} / \mathrm{c})^{2}$ using MSTW08L PDF, with LO DSS fit as well as with LUND string fragmentation model. In the Lund MC, MSTW08L was used, DSS used MRST04L $((u+d) /(\bar{u}+\bar{d})=2.05)$. A more recent PDF set from 2014 NNPDF3.0L predicts a lower $(u+d) /(\bar{u}+d)$ ratio of $1.85 \pm 0.10$, while earlier work of this group from NNPDF2.3 predicted the ratio of $2.30 \pm 0.20$. The newest NLO DSS fit predicts ratio $K^{+} / K^{-} 2.1$ for $z$ bin $0.75<$ $z<0.85$. Privately LSS reported that their NLO fits gives the ratio of 2.23 for $z=0.8$ and selected $x, Q^{2}$.

Kaon multiplicity ratio at high $z$.

## 3 Data Selection

The data selection follows the logic of [1], except for the differences discussed in the introduction, namely, the extended $z$ range and stricter cuts on the RICH PID. All 2006 data but W35 period were used and since the DIS sample is not needed to study the ratio all triggers were used in order to increase statistics. NOTE that in the current release we concentrate only on the region $x<0.05$. Results in the full phase space will be asked for the release for the paper.

- inclusive cuts
- Best PV is used
- $\mu$ and $\mu^{\prime}$ are reconstructed
- beam momentum in the range $140 \mathrm{GeV} / \mathrm{c}-180 \mathrm{GeV} / \mathrm{c}$
- beam crosses the whole target to equalise flux
- BMS vs beam telescope matching (if(Mu0.Chi2CutFlag()) return; )
$--56<P V_{z}<-35$ or $-20<P V_{z}<31$ or $43<P V_{z}<66$ (in cm)
- PV is in target in transverse plane (PaAlgo::InTarget used)
$-Q^{2}>1(\mathrm{GeV} / \mathrm{c})^{2}$,
$-0.004<x_{B j}<0.4$
$-0.1<y<0.7$
$-5 \mathrm{GeV} / \mathrm{c}^{2}<W<17 \mathrm{GeV} / \mathrm{c}^{2}$
- There is a physics trigger $((e . \operatorname{TrigMask}() \& 0 x 71 f)!=0)$, bad region of MT was removed
- hadron cuts
$-\operatorname{not} \mu^{\prime}$
- track has momentum and does not cross SM2 yoke
$-X X 0<15$
$-z_{\text {first }}<400$ and $z_{\text {last }}>400$
$-0.01<\theta_{\text {RICH }}<0.12$ - track angle at the RICH entrance
$-x_{\text {RICH }}^{2}+y_{\text {RICH }}^{2}>25 \mathrm{~cm}^{2}$ - track position at the RICH entrance
- Kaon definition:
* $L_{K}$ is the highest likelihood
* $L_{K} / L_{b c g r .}>2.08$
* $L_{K} / L_{2 n d}>1.5$ (original cut 1.08)

The final sample of event contains about 64000 kaons in the region $z>0.75$. Details concerning sample size after consecutive cuts are presented in Tables 1, 2. The kinematic distributions of $x, \mathrm{Q}^{2}, \nu$ and $z$ for the selected sample in the bin $x<0.05$ are presented in Fig 2

| $\#$ | events | ratio to previous | cut type |
| :---: | :---: | :---: | :---: |
| 1 | $2.41858 \mathrm{e}+09$ | - | before cuts |
| 2 | $2.24630 \mathrm{e}+09$ | 0.929 | PV exists |
| 3 | $2.24630 \mathrm{e}+09$ | 1.000 | $\mu$ in PV |
| 4 | $1.26299 \mathrm{e}+09$ | 0.562 | $\mu^{\prime}$ in PV |
| 5 | $8.82899 \mathrm{e}+08$ | 0.699 | BMS compatibility |
| 6 | $8.82883 \mathrm{e}+08$ | 1.000 | $140 \mathrm{GeV} / c<p_{\mu}<180 \mathrm{GeV} / c$ |
| 7 | $5.53071 \mathrm{e}+08$ | 0.626 | $P V_{z}$ cuts |
| 8 | $4.76125 \mathrm{e}+08$ | 0.861 | In target data |
| 9 | $4.75479 \mathrm{e}+08$ | 0.999 | In target MC |
| 10 | $4.68003 \mathrm{e}+08$ | 0.984 | Cross-Cell data |
| 11 | $4.66320 \mathrm{e}+08$ | 0.996 | Cross-Cell MC |
| 12 | $4.65433 \mathrm{e}+08$ | 0.998 | physics trigger |
| 13 | $4.74848 \mathrm{e}+07$ | 0.102 | $Q^{2}>1(\mathrm{GeV} / \mathrm{c})^{2}$ |
| 14 | $2.89479 \mathrm{e}+07$ | 0.610 | $0.1<y<0.7$ |
| 16 | $2.87843 \mathrm{e}+07$ | 0.994 | $5 \mathrm{GeV} / \mathrm{c}^{2}<W<17 \mathrm{GeV} / \mathrm{c}^{2}$ |
| 17 | $2.87515 \mathrm{e}+07$ | 0.999 | $0.004<x<0.4$ |

Table 1: Cut-by-cut reduction of events.

| $\#$ | events | ratio to previous | cut type |
| :---: | :---: | :---: | :---: |
| 1 | $1.17663 \mathrm{e}+08$ | - | before cuts |
| 2 | $8.89119 \mathrm{e}+07$ | 0.756 | not $\mu^{\prime}$ |
| 3 | $8.89119 \mathrm{e}+07$ | 1.000 | has momentum |
| 4 | $8.60025 \mathrm{e}+07$ | 0.967 | $z_{\text {first }}<400 \& z_{\text {last }}>400$ |
| 5 | $8.57631 \mathrm{e}+07$ | 0.997 | $X X 0>15$ |
| 6 | $8.43589 \mathrm{e}+07$ | 0.984 | $\theta_{R I C H}>0.01$ |
| 7 | $4.76273 \mathrm{e}+07$ | 0.565 | $\theta_{R I C H}<0.12$ |
| 8 | $4.69760 \mathrm{e}+07$ | 0.986 | $x_{R I C H}^{2}+y_{R I C H}^{2}<25 \mathrm{~cm}^{2}$ |
| 9 | $3.37136 \mathrm{e}+06$ | 0.072 | $z>0.5$ |
| 10 | $4.07683 \mathrm{e}+05$ | 0.129 | $\mathrm{PID=kaon,L}_{K} / L_{2 n d}>1.5$ |
| 11 | $3.70599 \mathrm{e}+05$ | 0.909 | $12 \mathrm{GeV} / \mathrm{c}<p<40 \mathrm{GeV} / \mathrm{c}$ |
| 12 | 64329 | 0.174 | $z>0.75$ |

Table 2: Cut-by-cut reduction of kaon candidates for the selected inclusive sample. The last 3 cuts are performed on the ROOT trees.

## 4 Analysis

The kaon multiplicity is defined as:

$$
\begin{equation*}
\frac{d M^{K}(x, y, z)}{d z}=\frac{1}{N^{D I S}(x, y)} \frac{d N^{K}(x, y, z)}{d z} \frac{1}{A(x, y, z)} \tag{4}
\end{equation*}
$$

In case of the multiplicity ratio there are some obvious cancellations, like $N^{D I S}$. Due to limited statistics the main analysis is performed in two $x$ bins. To better equalise statistics, instead of bins in $y$, a binning in the momentum of the observed hadron is done. Note, that since only the high $z$ region is studied there is a quite strong correlation between observed hadron momentum and $y$ and $\nu$.
In order to obtain the multiplicity ratio the experimentally measured ratio of $K^{+} / K^{-}$has to be corrected for RICH purity/efficiency as well as by the acceptance ratio for the two kaon charges. These two corrections are discussed below.


Figure 2: Kinematic distributions of $x, \mathrm{Q}^{2}, \nu$ and $z$ in the bin $x<0.05$.

### 4.1 RICH unfolding

The procedure of the RICH unfolding is well established and used since many years in various analyses. However, the systematic studies performed in view of this analysis revealed a likelihood behaviour in the high $z$ region which is not as expected. A large contamination of kaon sample by pions was detected. The details of this studies were presented in the analysis meeting in October, they are also briefly summarised in appendix A.
The conclusion from these studies is that one cannot use the unfolding method with present matrices in the high $z$ region. The predicted contamination of the kaon sample by pions is too low. There is no $100 \%$ reliable source of $\pi$ and $K$ at high $z$, which allows for proper calculation of the purity-efficiency matrix.
To avoid the observed $\pi$ contamination in the kaon sample (compared to the analysis [1]) the likelihood of kaon w.r.t. 2 nd highest $\left(L_{K} / L_{2 n d}\right)$ must be larger than 1.5 instead of 1.08 that was previously used. Otherwise all RICH cuts were kept the same as in published analysis. The value of the new cut was selected based on the observed $K^{+} / K^{-}$ratio as a function of $L_{K} / L_{2 n d}$, cf. Fig. 3. A plateau around $L_{K} / L_{2 n d}>1.4-1.5$ is reached.
The absence of a pion contamination was verified in two ways. In the first test the $p_{T}^{2}$ spectrum of the selected kaons was investigated. It could be fitted by a straight line, while with old


Figure 3: The ratio $K^{+} / K^{-}$as a function of RICH $L_{K} / L_{2 n d}$ cut. As the working point, marked by arrow, $L_{K} / L_{2 \text { nd }}>1.5$ value was selected. Following kinematic cuts were used: $x<0.05$, $\nu>30 \mathrm{GeV}, z>0.75$ and $30(\mathrm{GeV} / \mathrm{c})<p<40(\mathrm{GeV} / \mathrm{c})$.
cuts a clear contamination of the sample by $\pi$ coming from $\rho$ decays is visible, see Fig. 4. In the second exercise the RICH likelihoods were not used but instead results on the angle of the Cherenkov ring from the $\chi^{2}$ fits were used. With enough statistics one can fit the spectra to obtain the number of $\pi, K$ and/or p. The obtained ratio of $K^{+} / K^{-}$in this type of analysis is in agreement with the one obtained with the increased likelihood cut. Therefore, one can safely neglect possible $K$ misidentification in the analysis. What is also important is that a possible $\pi$ contamination would decrease the ratio of $K^{+} / K^{-}$. Thus it would bring the ratio closer to the prediction.

Formally the results should be corrected by the ratio of $K^{+} / K^{-}$RICH identification efficiency. However, there is no sample on which one can investigate high $z$ kaons to determine RICH efficiency with new likelihood cuts. What is clear that in the moderate $z$ region where efficiencies were extracted, they are very similar for the two charges. The average efficiency ratio is about $1.002 \pm 0.002$, the typical RMS in a given momentum and theta bin is about 0.012 . Here, two bins where the uncertainty of the efficiency extraction is a $3-5 \%$ were not counted. Since the problem with RICH likelihood affects positive and negative particles in the same way, there is no reason to assume that the ratio of efficiencies for RICH PID is different from 1 when higher likelihood cuts are used. Therefore in the analysis it is assumed that the efficiency ratio is equal to 1 , and a conservative limit of $3 \%$ is taken into account in the systematic uncertainty.

### 4.2 Acceptance Correction

In the previous analyses a rather straight forward method of acceptance correction was used, namely in a given $(x, y, z)$ bin one calculated the ratio of reconstructed and generated hadrons using reconstructed or generated variables, respectively. However, such a method has certain


Figure 4: Comparison of the $p_{T}^{2}$ spectra of hadrons with momenta $35-40 \mathrm{GeV} / \mathrm{c}$ which are identified as pions and kaons (left and centre panels) as well as kaon spectrum with increased likelihood cuts. In case of $\pi$ a clear contamination of the $\pi$ by decay products of $\rho^{0}$ is visible at low momenta. With standard RICH cuts a clear $\pi$ contamination is also visible in the kaon sample. The $\pi$ pollution is effectively removed by increased likelihood cuts $L_{K} / L_{2 n d}$ as seen on the right panel.
well-known drawbacks. Especially, since multiplicities strongly depend upon $z$ a investigation of the multiplicities at the edge of the kinematic region using such a method is not optimal and is MC dependent.
In order to take into account smearing in $z$ correctly, the acceptance ratio was calculated using the simple method for all variables but $z$. To do so, one extracts the acceptance as before, but for $z$ one does not use the reconstructed $z$ value but the generated ones. The acceptance ratio obtained in this way corrects for all effects but $z$ smearing. The obtained results as a function of $z_{g e n}$ are presented in Fig. 5 Within statistical uncertainties the acceptance is flat ${ }^{1}$.
The values is indeed much larger than 1.0. This is expected since $K^{-}$re-interact more frequently with the target material. The effect of $z$ smearing was studied using three methods. They are briefly described below.

### 4.2.1 The matrix method

In this method MC sample is used to build a smearing matrix in the phase space of $z_{\text {gen }}$ vs $z_{\text {rec }}$. The matrix does not have to be quadratic. For example, while $z_{\text {gen }}$ are restricted to $z_{\text {gen }}<1$, $z_{\text {rec }}$ can be larger 1.0, and thus additional rows in the matrix may appear. A vector $N_{\text {unfolded }}^{K}$ which are fit parameters is multiplied by such a matrix. The results are compared with the observed kaon yields and a standard $\chi^{2}$ can be calculated. The parameters are then determined using a minimisation procedure e.g. MINUIT. Note, that in case one would discard data with

[^0]

Figure 5: Acceptance ratio for $K^{+} / K^{-}$as a function of $z_{\text {gen }}$.
$z_{\text {rec }}>1$ and build a square smearing matrix, one has an analytic solution using just linear algebra.
The method has a large drawback, Namely, the uncertainties of $N_{u n f o l d e d}^{K}$ are rather large. But here are large negative off-diagonal elements present in covariance matrix. Unfortunately, since theory colleagues like DSS group, constantly ignore any covariance matrix, even if they were told to use them, using such a method of data unfolding means that our errors would be increased by a factor of $2-3$ in their analysis. While the results of using this method will be presented later, it is not the method of choice. The smearing matrix and the covariance matrix are presented later in appendix C.

### 4.2.2 Fit using functional form

In this method one obtains from the MC sample $z_{\text {rec }}-z_{\text {gen }}$ as a function of $z_{\text {gen }}$ (shown in appendix C). A functional form which is supposed to describe the unfolded data is convoluted with the extracted smearing from MC . Then the resulting function compared to the data and again a simple $\chi^{2}$ can be calculated and minimised. The main drawback of this method is the fact that convolution of a function with the smearing has to be done numerically after each step in MINUIT. The experience has shown that this might be a tricky point which may lead to some instabilities.

Namely, in the October analysis meeting the results of unfolding using such a method were presented, cf. slide 14,15 of [5]. One of the conclusions of M.S. at that time was that there is a clear difference observed at the edge of the spectra. Namely, the $K^{-}$spectrum ends earlier, while the $K^{+}$spectrum continues to higher $z$ values. While the result is compatible with LUND MC, further test have shown that $\chi^{2}$ is rather unstable and the quoted errors can be wrong by
a factor of about $3-4$. Thus the conclusion was that the observation of a different behaviour of $K^{-}$and $K^{+}$while approaching $z \rightarrow 1$ is premature at the moment.
However, the method has a large benefit, namely it is clear that nature follows rather smooth behaviour. It is not expected that the ratio of $K^{+} / K^{-}$will jump around for neighbouring z bins, rather a constant trend is expected. Such a trend is not guaranteed in the method described earlier, but it is in the current one.

### 4.2.3 The hybrid method

To benefit from the strong points of the two methods and eliminate weak ones a hybrid method was created. A functional form will be used but instead of a numerical convolution of the selected function with $z$ smearing obtained from MC the smearing matrix will be used.
The most robust solution was obtained in case a fitted functional form was integrated within the given $z$ bin limits and then multiplied by the smearing matrix. Again a simple $\chi^{2}$ can be calculated and minimised.

The method has a small drawback, namely the fit works in slightly reduced $z$ range. The point is that wanting to fit data with $z>0.80$ one has to take into account smearing from the bin $0.75-0.80$ into the bin with $z>0.8$. Yet as it will be shown, in COMPASS we can obtain a very reasonable fit to our data in the range $0.65<z<1.10$ using just one additional bin in $z$, $0.60<z<0.65$.

The obtained $10 x 8$ smearing matrix is presented in appendix C . The optimal functional form, motivated by the requirement that the multiplicity should be 0 for $z=1$, was found to be

$$
\begin{equation*}
\alpha \cdot \exp (-\beta z)(1-z)^{\gamma} \tag{5}
\end{equation*}
$$

In fact a $\chi^{2}$ better by a few units $(4-5)$ was obtained in case of a simpler fit: $\alpha \cdot \exp (-\beta z)+c$. However, allowing for non-zero multiplicity at $\mathrm{z}=1$ is not welcomed by the theory colleagues.

### 4.3 Further systematic studies

Comparing to the analysis presented in [1], here all 2006 data is used and in addition all triggers are used and not only the inclusive ones. Therefore, it is natural to verify if the extracted ratios are consistent with each other. It was found that for the data used in the previous analysis the average $K^{+} / K^{-}$multiplicity ratio for $x_{B j}<0.05$ and $0.7<z<1.1$ is $3.06 \pm 0.04$ and $3.07 \pm 0.06$ in added periods of 2006. Therefore, a very good agreement is obtained. As it will be described later there are possible problems observed in the low $y$ region. For safety, it was also verified that kaon multiplicity ratio in low $y$ region for the old and new data sample agree with each other. Namely, the ratio in the aforementioned $x$ and $z$ and $y<0.15$ is $4.26 \pm 0.12$ and $4.08 \pm 0.17$, for old and new data, respectively.
Trigger compatibility studies were also performed. A very good agreement between inclusive and semi inclusive triggers was found, namely a ratio of $2.96 \pm 0.05$ and $2.98 \pm 0.06$ for inclusive and semi inclusive triggers, respectively. However, a large discrepancy was observed for the pure calorimeter trigger, where the ratio is $3.38 \pm 0.08$. Further studies revealed that the observed problem is not related to the trigger itself but to the coverage of the phase space in hadron
$\phi$ defined in the laboratory system. Namely, pure CT covers mostly the region around $\phi=0$ i.e. Jura side, while other triggers cover whole region of $\phi$ with a hole around $\phi=0$. It was verified that for $|\phi|<\pi / 4$ inclusive and semi-inclusive triggers the kaon multiplicity ratio of $3.58 \pm 0.16$ is consistent with pure calo trigger result $3.79 \pm 0.14$. Therefore, further studies were performed to investigate stability of results as a function of $\phi$.

### 4.3.1 $\phi$ studies

In order to study in more details the $\phi$ dependence the region of $z>0.5$ was used and no RICH PID was used. It was verified that very similar problems are observed for hadrons, $\pi$ and $K$.
In the course of analysis it was verified that the $\phi$ spectra both for $h^{+}$and $h^{-}$are affected. The possible systematic problem affects more $h^{+}$and $h^{-}$distributions itself than their ratio. Thus, our published results for $\pi$ and $K$ multiplicities can be affected by this problem. An example of the problem is presented in Fig. 6 and Fig. 7. Figure 6 shows events yields as a function of $\phi$, i.e. $\phi$ angle of hadron measured in lab system at the position of PV, $\phi=0$ corresponds to Jura. The two rows correspond to $h^{+}$and $h^{-}$, respectively, In columns one has: reconstructed data, reconstructed MC and generated MC respectively. Using the MC distributions presented in Fig. 6, the acceptance corrected yield of $h^{+}$and $h^{-}$is presented in top-left and top-right panels of Fig. 7 The observed behaviour is not flat. In the bottom-left panel of Fig. 7 the $h^{+} / h^{-}$ratio is shown.

It should be stressed that quite some systematic effects which can generate a $\phi$ dependence cancel when integrated over $\phi$. In order to assure the best possible cancellation the distribution in $\phi$ should as flat as possible. Therefore, pCT was used in the analysis to fill a gap around $\phi=0$ present in acceptance of (semi)-inclusive triggers.
In the course of the studies it was verified that possible physics asymmetries in $\phi_{h}$ angle (i.e. $\gamma^{*}$ system angle measured w.r.t. lepton scattering plane, like Cahn effect) cannot explain the effect visible in $\phi$. It was verified that the problem is mostly pronounced at low values of $y<0.2$. Also a dependence with respect to the polar angle $\theta$ measured in laboratory frame was observed. For low values of $\theta$ the $\phi$ distribution is flat, while in the region $0.04<\theta<0.08$ instabilities are observed. Some plots are added in the appendix B.

So far there are not final conclusions concerning origin of the observed problem. Therefore, for the systematic uncertainty the difference is taken between results obtained in the whole $\phi$ phase space and with removal of events with low values of $\phi$, i.e. $|\phi|<\pi / 4$. The typical systematic uncertainty is between 1.4-1.8 of $\sigma_{\text {stat }}$.

### 4.3.2 $\pi$ multiplicity ratio

Because of the large contamination of the high $z$ region by decay products of $\rho^{0}$ mesons one cannot easily obtain interesting physics results from this region in case of $\pi$ analysis. However, studies of $\pi$ are important from point of view of systematic effects.
The observed acceptance and multiplicity ratio of $\pi^{+} / \pi^{-}$is presented in left and right panels of Fig. 8, respectively. From the preliminary LSS fit to COMPASS data at $<z>=0.8$, $\left\langle x_{B j}\right\rangle=0.037$ and $Q^{2}=1.8(\mathrm{GeV} / \mathrm{c})^{2}$ one would expect a pion ratio of about 1.26. Taking into account that the presented data were not corrected for a VM contribution, which can be


Figure 6: In rows yields of $h^{+}$and $h^{-}$are given, respectively. The tree columns corresponds to: data, reconstructed MC, generated MC. The event yields are shown as a function of $\phi$ measured in the laboratory system as measured in PV.
as large as $25-30 \%$ for $z=0.8$ and increase up to about $40-60 \%$ for higher $z$, one expects a ratio between 1.20-1.18 at $z=0.80$ and 1.15-1.10 at high $z$. Thus, taking into account the rather large uncertainty of the VM correction the presented results agree reasonably well with the expectations from NLO fits.
One should stress one more important point. It is consensual than ratio of $\pi^{+} / \pi^{-}$must be larger than 1.0. Since we observe a ratio of about 1.1 at high $z$, it is clear that a possible bias due to the $\phi$ dependence discussed in previous section cannot be larger than $10 \%$ as otherwise nonphysical values of $\pi^{+} / \pi^{-}<1$ would be obtained.

## 5 Results and Discussion

The obtained ratio in the range $z>0.75$ for $x<0.05$ is presented in Fig. 9. The ratio is corrected for the acceptance difference between $K^{+} / K^{-}$but not for the $z$ smearing, which will be discussed later. However, since multiplicities are decreasing with an increase of $z$ the smearing can only further increase the obtained ratio. The measured ratio of $K^{+} / K^{-}$is therefore much higher than the expectation from LO QCD. Similarly the large discrepancy between data and LUND MC as well as the DSS LO QCD fit are visible.
Further studies presented in Fig 10 reveal that at least in the $z$ range $0.75-0.95$ there is a clear


Figure 7: In top row the acceptance corrected yields $h^{+}$(left), $h^{-}$(right) are presented as a function of $\phi$. In the bottom row the ratio $h^{+} / h^{-}$is shown.
$\nu$ dependence of the obtained ratio. With increasing $\nu$ the ratio is getting closer to the limit of pQCD, but it is quite far away from it for lowest $\nu$ values. Therefore, it would be expected that in case of HERMES even larger ratios could be potentially observed. In the HERMES published range $0.2<z<0.8$ indeed the HERMES $K^{+} / K^{-}$ratio is always larger than the COMPASS one (contrary to the $\pi$ case where two results agree).

The observed large kaon multiplicity ratio explains the outcome of our LO pQCD fit of fragmentation functions that is contained in a preliminary release, which was shown on conferences. However, just before the publication it turned out that the results can be significantly improved (a factor 1.5 in $\chi^{2} / n d f$ ) in case a negative value of $D_{u n f}$ is allowed at high $z$. In fact the same observation can be done in a NLO analysis, where $\chi^{2} / n d f$ can be reduced by a factor of about 1.3 by also allowing negative $D_{u n f}$ at the reference scale. Now, it is clear that negative $D_{u n f}$ is a fit way to compensate for the large ratio of $K^{+} / K^{-}$seen in data which is further confirmed at high $z$ in this analysis.
While the presented limit $M^{K^{+}} / M^{K^{-}}<(u+d) /(\bar{u}+\bar{d})$ may be violated in NLO, it is clear that the large improvement of $\chi^{2} / n d f$ shows that the data cannot be described well by using standard pQCD formalism at NLO. Note that for years the unpolarised $g$ PDF was negative in NLO fit, thus at least to some colleagues negative $D_{u n f}$ value at the reference scale may be acceptable. The problems with the description of the COMPASS data at low $y$ and high $z$ were also recently confirmed by LSS group. Their observations fit well to what we see in the


Figure 8: Left Panel: Acceptance ratio for $\pi^{/} \pi^{-}$as a function of $z_{r e c}$, Right Panel: Multiplicity ratio of $\pi^{/} \pi^{-}$as a a function of $z_{\text {rec }}$.
present analysis as well as in the results of NLO fits using non-negative results of $D_{\text {unf }}$. In case a negative $D_{u n f}$ is not acceptable, our results may mean that the factorisation and universality is broken in the high $z$ region. Suppose that in case of $K^{-}$the only allowed final states are $X$, in case of $K^{+}$one can also produce $\mu p \rightarrow \Lambda K^{+}$. If factorisation holds we have first interaction with $u$ or $\bar{u}$ quark, and then in the second step the final state is chosen for $K^{-}$among X and for $K^{+}$among $\bar{X}$ and $\Lambda$. The observed limit for the kaon ratio is still $u / \bar{u}$ for proton target. However, if factorisation is broken one can imagine a picture that all possible final states are considered "prior to interaction", thus one would produce $X$ states according to the ratio $u / \bar{u}$ for both $K^{+}$and $K^{-}$, but additional $K^{+}$would be produced because of open phase space for $K^{+} \Lambda$ production. In such a case also universality is broken, since in $e^{+} e^{-}, K^{+} \Lambda$ vs $K^{-} \bar{\Lambda}$ is symmetric.
So far the presented results did not take into account $z$ smearing. The results presented in top panel of Fig. 11, were obtained using the matrix method described in section 4.2. In the fit $\chi^{2} / n d f=8.95 / 4$ is obtained. However, at a given $z$ point the uncertainties of the unfolded data are much larger than what is obtained in raw data. One should stress that it is expected that unfolded data are larger than the measurement at a given point. As already discussed the larger error bars are in fact not problematic in case the covariance matrix is properly taken into account. Unfortunately, such a practise is not common among theory groups.
However, much nicer results can be obtained using the hybrid method. The results are presented in the bottom panel of Fig. 11, the errors calculation is using the bootstrap method. The obtained $\chi^{2} / n d f$ is $19.93 / 12$, the $\chi^{2}$ probability is about $7 \%$. The presented fit was performed starting from $z=0.65$. This shows that the assumed functional from can be extended into a


Figure 9: COMPASS multiplicity ratio of $K^{+} / K^{-}$for $\langle x\rangle=0.03$ and $\left\langle Q^{2}\right\rangle=1.6(\mathrm{GeV} / \mathrm{c})^{2}$, and its comparison with LO QCD limit $(u+d) /(\bar{u}+\bar{d})$, LUND MC and DSS LO fit. A PDF set MSTW08L was used for calculation of LO QCD limit as well as in MC production, DSS used MRST04L $((u+d) /(\bar{u}+\bar{d})=2.05$ in their FF fit. A more recent PDF set from 2014 NNPDF3.0L predicts lower $(u+d) /(\bar{u}+\bar{d})$, ratio 1.9 , while earlier work of this group from 2012 predicted the ratio of 2.35 .
larger region, and a reasonable fit is obtained ${ }^{2}$. The obtained functional forms are the following:

$$
\begin{aligned}
& N K^{+}=6.52 \cdot 10^{5} \exp (-4.693 z)(1-z)^{0.3} \\
& N K^{-}=4.71 \cdot 10^{5} \exp (-4.197 z)(1-z)^{1.0} .
\end{aligned}
$$

For given $z$ they estimate the number of observed events in the range $(z-0.025, z+0.025)$. To obtain proper a $K^{+} / K^{-}$ratio the acceptance ratio of 1.086 has to be taken into account.
The fit can be further improved by assuming that the $K^{-}$spectrum does not end at $z=1$ but around $z=0.973$ (gain in $\chi^{2}$ by 5.4 units), also a simpler functional, which is not bound to be zero at $z=1$, gives a somewhat better $\chi^{2}$. The best obtained fit so far had $\chi^{2} / n d f \approx 12 / 10$, but it could be seen as controversial by some theory colleagues.

[^1]

Figure 10: COMPASS multiplicity ratio of $K^{+} / K^{-}$as a function of $\nu$ in bins of $z$. In all but highest $z$ bin a clear $\nu$ dependence of the ratio is observed

## 6 Summary and Outlook

The ratio of $K^{+} / K^{-}$were presented in the high $z$ region. The trend observed in Fig. 7 of [1] is confirmed. The obtained results are much higher than expectation of (N)LO pQCD. COMPASS should aim to publish as soon as possible these interesting results.
The largest contribution to the systematic uncertainty comes from the observed $\phi$ instability. It is possible that with further studies these uncertainty could be reduced, however the physics conclusions will not be altered. Therefore, most probably it is more beneficial to finish 2006 analysis as soon as possible and concentrate on 2016/17 data. There are several advantages of doing so. First of all, because of $\mu^{+}$and $\mu^{-}$beams and as a consequence two spectrometer magnets polarities in 2017 data the observed $\phi$ dependence will cancel in the ratio of $K^{+} / K^{-}$ multiplicity. The proton target is a simpler system, where nuclear effects do not play any role. In the present analysis there are a lot of events where a single $K^{+}$is observed in the spectrometer. It is possible that the excess of $K^{+}$is due to production $\mu p \rightarrow K^{+} \Lambda^{0}$. While some further studies are being performed on 2006 deuteron data, the presence of the recoil proton detector in 2016/2017 data will allow for more detailed studies of the interaction region. Finally, software-wise improvement of RICH for the old 2006 data are hard to be justified. Yet, the situation is different for new the 2016-17 data. Especially, with better RICH performance one could extend the range of momenta where studies are performed. Thus, study a region of higher $\nu$, where the results seem to be closer to pQCD predictions.


Figure 11: Unfolded ratio of $K^{+} / K^{-}$using two methods: matrix method (top) and hybrid method (bottom).

## 7 Cross-Check

The cross-check was preformed between Marcin and Nicolas. It should be noted that since both of us were involved in previous multiplicity analyses we already had independent codes almost cross-checked. Therefore, it was decided that the cross-check of the data selection is performed on 1 period and MC was tested on 100k events. After these cross-check M.S produced remaining the data periods and MC files, and the cross-check was continued on such pre-selected sample. The results of the two analyses agree as presented in Tables 3, 4 and Figs. 1213

## References

[1] COMPASS, C. Adolph et al., PLB 767 (2017) 133.
[2] COMPASS, C. Adolph et al., EPJC 72 (2012) 2253.

| $\#$ | Marcin | Nicolas | cut type |
| :---: | :---: | :---: | :---: |
| 1 | 157584729 | 157584729 | before cuts |
| 2 | 147859920 | 147859920 | PV exists |
| 3 | 147859920 | 147859920 | $\mu$ in PV |
| 4 | 84680007 | 84679722 | $\mu^{\prime}$ in PV |
| 5 | 55948505 | 55948289 | BMS compatibility |
| 6 | 55947497 | 55947282 | $140 \mathrm{GeV} / c<p_{\mu}<180 \mathrm{GeV} / c$ |
| 7 | 34851567 | 34851431 | $P V_{z}$ cuts |
| 8 | 30128158 | 30128040 | In target data |
| 9 | 30087511 | 30087393 | In target MC |
| 10 | 29622481 | 29622364 | Cross-Cell data |
| 11 | 29514951 | 29514834 | Cross-Cell MC |
| 12 | 29468766 | 29468655 | physics trigger |
| 13 | 2832334 | 2832362 | $Q^{2}>1(\mathrm{GeV} / \mathrm{c})^{2}$ |
| 14 | 1749475 | 1749455 | $0.1<y<0.7$ |
| 15 | 1739476 | 1739456 | $5 \mathrm{GeV} / \mathrm{c}^{2}<W<17 \mathrm{GeV} / \mathrm{c}^{2}$ |
| 16 | 1737414 | 1737393 | $0.004<x<0.4$ |

Table 3: Result of cross-check on one week of data.

| $\#$ | Marcin | Nicolas | cut type |
| :---: | :---: | :---: | :---: |
| 1 | 7166205 | 7166174 | before cuts |
| 2 | 5428791 | 5428781 | not $\mu^{\prime}$ |
| 3 | 5245395 | 5245385 | $z_{\text {first }}<400 \& z_{\text {last }}>400$ |
| 4 | 5230425 | 5230415 | $X X 0>15$ |
| 5 | 2891864 | 2891868 | $0.01<t h_{R I C H}<0.12$ |
| 6 | 2852807 | 2852812 | $x_{R I C H}^{2}+y_{R I C H}^{2}<25 \mathrm{~cm}^{2}$ |
| 7 | 128887 | 128889 | PID=kaon, $L_{K} / L_{2 n d}>1.5$ |

Table 4: Cross-check results concerning number of kaons in the sample (no $z$ cuts), one week of data.


Figure 12: Cross-check results MS (red) NdFvH (blue).
[3] M. Stolarski, RICH PID: puzzles in $\pi /$ K-ID at higher (35-40 GeV) momenta, Analysis meeting October 2016
[4] M. Wilfert, Another view at RICH PID puzzles, Analysis meetingOctober 2016

Kaon multiplicity ratio at high $z$.


Figure 13: Cross-check results MS (red) NdFvH (blue).
[5] M. Stolarski, $K^{-} / K^{+}$Multiplicity ratio at high z, Analysis meeting October 2016

## A RICH problem

A few more slides are presented concerning the RICH problems. Various instabilities, which after some work can be corrected offline, are shown. However, the bottom line is that with increased likelihoods cuts these worrisome effects do not bias presented results of kaon multiplicity ratio.



Figure 14: Likelihood of $\pi$ and $K$; Left panel before cut, Right panel after cuts. $\pi$ contamination to the $K$ sample is clearly visible.


Figure 15: Left panel: $\theta$ of Cherenkov ring in the likelihood fit as a function of Run. Run by run instabilities are observed. Right panel: The same but for the $\theta$ from the $\chi^{2}$ fit after run-by-run correction.


Figure 16: Left panel: Comparison of $\theta_{\text {ring }}$ estimation before and after run-by-run correction. The difference in RMS is $600 \mu \mathrm{~m}$ vs $400 \mu \mathrm{~m}$; Right panel: The $\theta_{\text {ring }}$ using $\chi^{2}$ fit method in some restricted area in the spectrometer. A resolution of about $250 \mu \mathrm{~m}$ is observed, much better than $400 \mu \mathrm{~m}$ observed on the left panel.


Figure 17: Left panel: an example of $\theta_{\text {ring }}$ as a function of x projection of $\theta$ angle at the RICH entrance for track with momentum about $20 \mathrm{GeV} / \mathrm{c}$. Right panel: Is a profile of the left plot, large instabilities are observed of the order of $700 \mu \mathrm{~m}$ thus almost 3 x more than local $250 \mu \mathrm{~m}$ resolution seen in previous figure.


Figure 18: A $\theta_{\text {ring }}$ vs track $x$ position at 25 m . The plot is made for higher momenta, 35-40 $\mathrm{GeV} / \mathrm{c}$ than the previous one. More detailed internal structure of instabilities is seen.


Figure 19: Left panel: For a given logarithm of the likelihood of the $\pi$ (red) and $K$ (blue) hypothesis is shown as a function of $\theta_{\text {ring }}$ reconstructed in the $\chi^{2}$ method. Unexpected double peak is seen for K. The effect of such a likelihood behaviour, can be seen in the right panel, here $\theta_{\text {ring }}$ as a function of hadron $\phi$ is shown for particles identified as kaons using standard procedure. A clear influx of $\pi$ to the kaon sample in around top and bottom $( \pm \pi / 2)$ is seen.


Figure 20: A $\theta_{\text {ring }}$ using $\chi^{2}$ method for all particles with momenta between $35-40 \mathrm{GeV} / \mathrm{c}$ (blue), the one identified as kaon using standard cuts (red), the one identified as kaons using in addition $L_{K} / L_{2 n d}>1.5$ (black). Additional studies have shown that the sample with the additional cut $L_{K} / L_{2 n d}>1.5$ is indeed a pure K sample. However, from the figure it is clear than such a method of sample selection is not optimal from statistical point of view. Moreover, at the moment using the $\chi^{2}$ method of the ring fit one can obtain better and more stable results. This points towards problems with the likelihood method which should be "the optimal one". One worrisome point, besides the observed instabilities in $\theta_{\text {ring }}$, is that in the current likelihood fit background photons are assumed to be uncorrelated. In reality this assumption is not true, as most of the "background photons" belongs to other rings, i.e. rings are overlapping.

## B $\phi$ angle problem



Figure 21: In rows yields of $K^{+}$and $K^{-}$are given, respectively. The tree columns corresponds to: data, reconstructed MC, generated MC, respectively. The event yields are shown as a function of $\phi$ measured in the laboratory system as measured in PV. Additional problems may appear due to RICH problem discussed earlier.


Figure 22: In top row the acceptance corrected yields $K^{+}$(left), $K^{-}$(right) are presented as a function of $\phi$. In the bottom row the ratio $K^{+} / K^{-}$is shown. The observed modulation seems to be similar or a bit smaller than for unidentified hadron case


Figure 23: As Fig. 7 but for $y<0.2$.


Figure 24: As Fig. 7, but for $y>0.3$.


Figure 25: As Fig. 7 but for $y<0.2$ and hadron polar angle in PV, $\theta<0.04$


Figure 26: As Fig. 7 but for $y<0.2$ and hadron polar angle in PV, $0.04<\theta<0.08$


Figure 27: As Fig. 7 but for $y<0.2$ and hadron polar angle in PV, $0.08<\theta<0.12$


Figure 28: Impact in $\phi$ of the physics modulation in $\phi_{h}$, i.e. the $\phi$ angle measured w.r.t. $\gamma^{*}$ axis, and lepton scattering plane. A $\cos \phi_{h}$ (left) and $\cos 2 \phi_{h}$ modulations were tested. An asymmetry of amplitude 0.2 was induced for both charges (red point) or only one charge (black points). Note, that typical physics asymmetry is about 0.05 in most of the kinematic region. Therefore, the physics asymmetries cannot explain magnitude of dependence observed in $\phi$.

## C The smearing matrix

The smearing matrix used in the unfolding procedure is presented in Table 5. There are 10 bins in $z_{\text {rec }}$ (rows) and 8 bins in $z_{\text {gen }}$ (columns). The first bin edge in both case is $z=0.6$ and the bin width is 0.05 . For $z_{\text {rec }}$ the last bin edge is 1.1 , and 1.0 for $z_{g e n}$. The typical relative uncertainty of $d z / z$ as a function of $z$ is presented in Figure 29.

| 0.6646 | 0.1920 | 0.0144 | 0.0029 | 0.0014 | 0.0007 | 0.0006 | 0.0003 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1492 | 0.6218 | 0.2102 | 0.0240 | 0.0049 | 0.0021 | 0.0009 | 0.0003 |
| 0.0085 | 0.1601 | 0.5804 | 0.2276 | 0.0340 | 0.0069 | 0.0030 | 0.0026 |
| 0.0012 | 0.0128 | 0.1733 | 0.5421 | 0.2412 | 0.0423 | 0.0122 | 0.0026 |
| 0.0003 | 0.0016 | 0.0162 | 0.1786 | 0.5080 | 0.2574 | 0.0582 | 0.0161 |
| 0.0001 | 0.0004 | 0.0020 | 0.0201 | 0.1807 | 0.4751 | 0.2673 | 0.0837 |
| 0.0000 | 0.0001 | 0.0005 | 0.0025 | 0.0245 | 0.1816 | 0.4445 | 0.2819 |
| 0.0000 | 0.0001 | 0.0003 | 0.0007 | 0.0035 | 0.0280 | 0.1766 | 0.4064 |
| 0.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0010 | 0.0046 | 0.0311 | 0.1706 |
| 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0008 | 0.0051 | 0.0357 |

Table 5: The smearing matrix (10x8) used in the unfolding procedure.

| 1.000 | -0.704 | 0.404 | -0.219 | 0.111 |
| ---: | ---: | ---: | ---: | ---: |
| -0.704 | 1.000 | -0.753 | 0.449 | -0.237 |
| 0.404 | -0.753 | 1.000 | -0.776 | 0.455 |
| -0.219 | 0.449 | -0.776 | 1.000 | -0.762 |
| 0.111 | -0.237 | 0.455 | -0.762 | 1.000 |

Table 6: Typical correlation matrix obtained in the unfolding procedure using the above smearing matrix. Note large negative correlations around the diagonal. Five $z$ bins in range ( $0.75-1.0$ ) are shown.


Figure 29: A $d z_{g e n} / z_{g e n}$ as a function of $z_{g e n}$. A rather smooth behaviour is observed, a fit by a straight line fits the data well.

## D Material for the Release

The current release is rather an "addendum" to the published kaon results. Therefore, for the moment very few plots are asked to be released. More plots will be requested on a later date to be put in the paper, e.g. basic kinematic distribution, acceptance, $\pi$ results, full release of current data set. The release request for the moment is:

- Number of $K^{+}$and $K^{-}$for $x<0.05$ and $z>0.75$ : about 40000
- $\langle x\rangle=0.03,\left\langle Q^{2}\right\rangle=1.6(\mathrm{GeV} / \mathrm{c})^{2}$
- average acceptance ratio: 1.086


Figure 30: Comparison of the QCD prediction of the $K^{+} / K^{-}$limit using MSTW08L PDF, with the multiplicity predictions from the LO DSS fit, as well as with the LUND string fragmentation model. In Lund MC MSTW08L was used, DSS used MRST04L.


Figure 31: COMPASS multiplicity ratio of $K^{+} / K^{-}$.


Figure 32: COMPASS multiplicity ratio of $K^{+} / K^{-}$, and its comparison with LO QCD limit, LUND MC and DSS LO fit. The PDF set MSTW08L was used for the calculation of the LO QCD limit as well as in the MC production; DSS used MRST04L in their FF fit.


Figure 33: COMPASS multiplicity ratio of $K^{+} / K^{-}$in two $z_{\text {rec }}$ bins as a function of $\nu$.


Figure 34: Unfolded $K^{+} / K^{-}$ratio using the hybrid method.


[^0]:    ${ }^{1}$ Thus, in fact it is safe to use the average acceptance value also in $z_{r e c}$ variable!

[^1]:    ${ }^{2}$ It is an improvement w.r.t. functional function fit method as presented in October analysis meeting, where the fit could be only performed for $z>0.75$ data.

